# APPENDIX C

# ERROR ANALYSIS

This appendix outlines the error analysis used for this work. The general treatment of errors is given in Section C.1, and a specific example of the error in the measurements of free surface velocity is treated in Section C.2.

## C.1. General Treatment of Errors

The error in a quantity is the uncertainty in its measurement. The value of a quantity may depend on two or more other parameters. Each of these parameters is uncertain to some degree. To estimate the total uncertainty in the quantity requires a prescription for calculating the error which includes the propagation of errors. The present error analysis gives such a prescription which depends on obtaining analytical expressions of the error in terms of known parameters. The analysis is based on a book by Yardley Beers.[63]

The errors are reported in the shorthand form "$f \pm \varepsilon$" where $\varepsilon$ is the average deviation of $f$. The total magnitude of the error is just $2\varepsilon$.

Errors may be random, as in the case when a measurement is repeated several times. The results will be distributed around a "most probable" value which is assumed here to be the average value. For random errors there is a possibility of compensation among the various contributions. It is expected that the total error will be algebraically less than the sum of the separate contributions. A logical way of adding the separate contributions is to take the square root of the sum of their squares (hereinafter called SRSS) which does have the compensating property. The rule for combining random errors is

$$\varepsilon_f = \left[\left(\frac{\partial f}{\partial x}\right)^2 \varepsilon_x^2 + \left(\frac{\partial f}{\partial y}\right)^2 \varepsilon_y^2\right]^{1/2} \qquad (C.1)$$

where $f = F(x,y)$ and $x,y$ are the independent measured parameters. The function $f$ can depend on any number of parameters. The choice of two parameters for Eq. (C.1) was made only for illustrative purposes.

Errors may be systematic where all the individual values are in error by the same amount. These errors cannot be estimated by repeated measurements. The systematic error often comes from the uncertainty in the calibration of the instrument used to measure the parameter. These errors combine algebraically because compensation among the various contributions is not likely. Therefore,

$$\varepsilon_f = \frac{\partial f}{\partial x}\,\varepsilon_x + \frac{\partial f}{\partial y}\,\varepsilon_y \,. \qquad (C.2)$$

When both kinds of errors are encountered they are combined by taking the SRSS of both separate contributions to give a total estimate of error for that particular parameter.